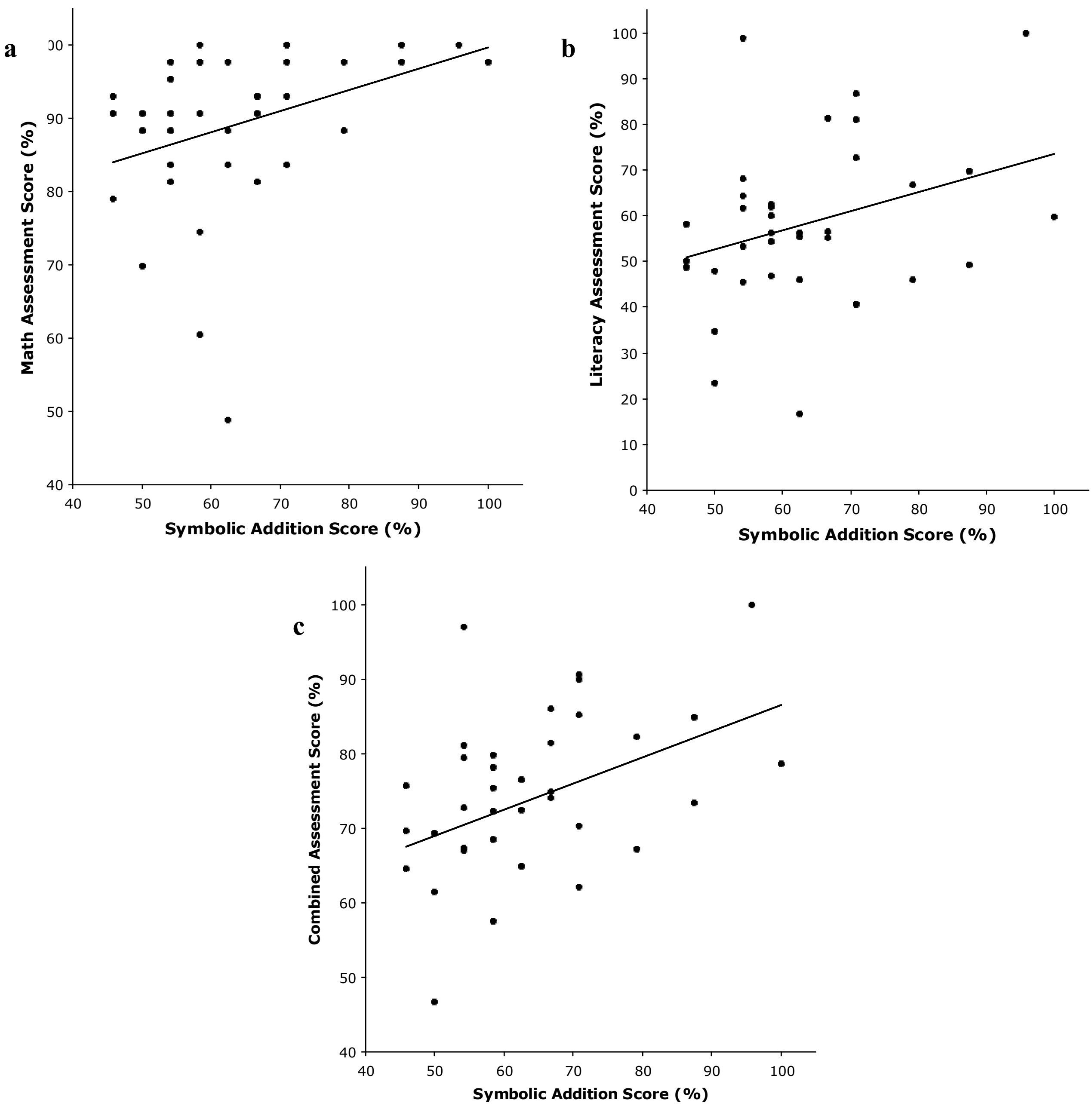
doi: 10.1038/nature05850

Correlation between nonsymbolic addition and school achievement.

Performance on nonsymbolic number tasks is associated with performance of exact, symbolic arithmetic both in adults and in older children. We asked, therefore, whether kindergarten children's relative success at approximate symbolic arithmetic is associated with mastery of their school's mathematics curriculum. The children who participated in the school-based study were given end-of-year achievement tests focusing on their learning of verbal and quantitative skills, which were created and administered independently by the school district. In contrast to our experiments, the quantitative test focused on small numbers. It consisted of 43 questions assessing basic kindergarten math skills such as counting, ordinal numbers, number lines, measurement, sets and graphs. No test item included quantities greater than 10 or required formal addition. The literacy test consisted of 323 brief questions testing awareness of rhymes, visual recognition of letters, generation of the sounds of letters, reading of high frequency words, and hearing and recording sounds in words. Each test was administered by the child's kindergarten teacher within a month of the time that our experiment was conducted; teachers were not aware of children's performance in that experiment. On the basis of the tests, each child was given a score of math achievement (percent correct responding on 43 items), reading achievement (percent correct responding on 323 items) and a combined score (the mean of the two scores). Children's performance of symbolic addition predicted their combined score on these two tests ($r = .373$, $p = .023$) as well as their performance on the quantitative test ($r = .343$, $p = .037$), and it was non-significantly associated with performance on the verbal test ($r = .313$, $p = .06$; Supplementary Figure 1). Thus, the capacity for symbolic, approximate arithmetic is related to measures of children's mastery of their kindergarten curriculum. Further research, including curricular intervention studies, is necessary to ascertain both the specificity and the causal nature of this relationship.



Supplementary Figure 1. Correlations between performance of symbolic approximate addition and performance on (a) a test of learning of exact, small numbers, (b) a test of learning of vocabulary, and (c) a combined measure of general school achievement. The tests were administered by teachers at the end of the school year.

Supplementary Analyses of Strategy Use

More /Less. Children who tended to answer only more/bigger or only less/smaller would score 50% overall and would show an asymmetry in performance for problems with the correct answer more/bigger or less/smaller. Because performance was above 50% in all experiments, this strategy cannot account for children's performance. Nevertheless, the children in the first and third addition experiment performed above chance for problems where the sum was more (respectively, 75.0%, t_{19} = 3.34, P = .003; 71.9%, t_{36} = 5.631, $P = 2 \times 10^{-6}$) but at chance for problems where the sum was less (respectively, 55.0%, t_{19} = 0.58, P = .569; 55.9%, t_{36} = 1.341, P = .188), consistent with a bias to judge the sum as larger. The children in the second addition experiment and in the subtraction experiment showed no such asymmetry and performed above chance both when the sum or difference was greater than the comparison number (respectively, 70.0%, t_{19} = 2.654, P = .016; 74.6%, t_{27} = 4.99, $P = 3 \times 10^{-5}$) and when it was the lesser quantity (respectively, 76.7%, t_{19} = 6.140, P = .016; 62.1%, t_{27} = 2.30, P = .029). Because no child in any experiment performed reliably below chance on problems for which the sum or difference was the lesser quantity, this simple comparison strategy cannot account for children's performance.

Exact arithmetic. To investigate whether children were able to solve the problems using exact arithmetic, a subset of children in the addition and subtraction experiments were asked to give the exact result for two questions that they had answered correctly during the forced choice task (e.g. "Sarah has 21 candies, she gets 30 more, how many does she have altogether?"). For most of the questions, the children did not know how to attempt the problems or quickly abandoned calculation efforts. The two children who provided a single correct response to an addition problem did so after a long effortful overt calculation process (counting on their fingers) and responded in an average of 11.5 seconds. In contrast, the average response time in the main task was only 1.9 seconds, well below the time 8-year-old children take to perform exact symbolic arithmetic (Barrouillet & Lepine, 2005). None of the children answered the exact subtraction questions correctly.

To investigate in more detail whether children could have solved these problems using exact arithmetic, a follow-up study was carried out. A new sample of 16 children aged 5 – 6 years (mean 5:7) was given the same 24 addition problems used in the computer-based addition experiments. Half of these problems at each ratio had the same comparison quantity as used in the previous experiments and therefore tested approximate addition. The other problems had a comparison quantity that differed by only 1 or 2 from the correct answer and thus required children to perform an exact comparison (e.g. approximate problem: 20 + 16 vs. 45; exact problem: 15 + 25 vs. 41). The children performed significantly above chance on the problems involving an approximate comparison (61%, $t_{15} = 3.00$, $P =$ .009) but no better than chance on the problems involving an exact comparison (49%, $t_{15} = 0.159$, $P =$ .876). Performance was significantly lower on the problems involving exact comparisons ($t_{15} = 2.54$, $P =$ .023).

Rounding strategy. We tested whether children solved the problems by performing small-number additions after rounding to decades. If children used a rounding strategy, performance should have been higher for problems with a round number in the addends. Separate analyses of the relevant subsets of problems in the computer based addition experiments revealed no difference in performance on problems with or without a round number (laboratory addition: 67.5% vs. 75.3%, $t_{19} = 1.38$, $P = .183$; classroom addition: 58.6% vs. 65.6%, $t_{36} = -1.678$, $P = .102$; subtraction: 68.3% vs. 64.7%, $t_{27} = .791$, $P =$ .436). In the verbal addition experiment, performance was significantly higher on problems that did not include a round number (68.8%) than problems that did include a round number (61.3%, $t_{19} = -2.349$, $P =$ .030). Thus, there is no evidence that children performed small-number addition after rounding to decades.

Single-digit arithmetic strategy. An alternative way to convert large- to small-number additions is to focus on single digits in the problems. For the comparison problems in the last experiment, children could compare the digits in the tens or units position, or choose the number that had the largest individual numeral. Children performed above chance on the comparison problems for which these three strategies give the incorrect answer (respectively, $t_{28} = 9.52$, $P = 3\text{x}10^{-10}$; $t_{28} = 6.34$, $P = 7\text{x}10^{-7}$; $t_{28} = 5.45$, $P = 8\text{x}10^{-6}$), providing evidence against use of these strategies. For the addition and subtraction problems, children could perform single digit arithmetic of the left-hand digit, or they could simply choose the number that has the largest individual numeral. Supplementary Tables 1 and 2 summarize performance on the subsets of problems for which these strategies fail to yield the correct answer. In all the experiments except the first, children performed above chance on these subsets of problems. In the first experiment, performance was in the correct direction. All these analyses provide evidence against use of single-digit comparison and calculation strategies.

Supplementary Table 1: Performance in each experiment on the subset of problems in which selecting the largest numeral predicts the incorrect response.

Experiment	Trials / subject	Accuracy	*P* Value	> Chance (50%)
1, addition	6	60.8	$P = .097$	No*
2, addition	16	73.8	$P = 2\times10^{-7}$	Yes
3, addition	16	67.7	$P = 2\times10^{-8}$	Yes
4, subtraction	10	65.4	$P = .002$	Yes

* Although performance on these trials is not significantly above chance, the mean score is above 50%. If children were using the largest digit strategy to solve these problems performance would be significantly below chance.

Supplementary Table 2: Performance in each experiment on the subset of problems for which considering only the left-hand digit does not provide a shortcut to the correct solution.

Experiment	Trials / subject	Accuracy	*P* Value	> Chance (50%)
1, addition	2	52.5	$P = .789$	No*
2, addition	10	79.5	$P = 2\times10^{-8}$	Yes
3, addition	10	70.8	$P = 1\times10^{-8}$	Yes
4, subtraction	9	67.9	$P = 3\times10^{-6}$	Yes

* The small number of problems for which the left-hand digit strategy cannot be used limits the extent to which this strategy can be tested in Experiment 1.

Range strategy. The next analysis tested whether children base their answers on the range of values tested across the set of trials: i.e., whether children judge that the comparison set is larger than the sum or difference whenever it is particularly large, or that the sum is smaller whenever one or both addends are particularly small. Each experiment presented two sets of 12 addition or subtraction problems with non-overlapping ranges of values, and all experimental variables were counterbalanced within each range. Use of any of these strategies therefore would predict the correct answer on half of the trials at each comparison ratio and the incorrect answer on the remaining trials. For example, on half of the trials when the comparison set was large (above the median) it was smaller than the sum of the addends (see Supplementary Table 5). Therefore, if children were using these strategies they would perform at chance overall and below chance on the subset of trials for which the size of the comparison quantity or the addends predicts the incorrect answer.

Because children performed above chance in all the experiments, this strategy does not fully account for their performance. To evaluate residual use of these range strategies, Supplementary Table 3 presents children's performance on the problem subsets for which the range strategy predicts the incorrect answer. There was no evidence that this strategy played a role in the three ratio addition experiments, for children performed above chance on problem subsets for which the size of the comparison array or addends predicts the incorrect response. In the first addition experiment and the subtraction experiment, children's performance was not significantly above chance on all of the critical subsets, but it also was not in the direction predicted by the range strategies. Because no child performed below chance on the subset of problems for which the range strategies predict the wrong answer, these strategies do not account for children's performance in any experiment.

Supplementary Table 3: Performance in each experiment on the subset of problems for which the range strategy predicts the incorrect answer.

Experiment	Trials / subject	Accuracy	*P* Value	> Chance (50%)
1, addition (addend)	3	63.3	*P* = .049	Yes
1, addition (comparison set)	3	56.7	*P* = .345	No*
2, addition	12	66.3	*P* = .007	Yes
3, addition	12	58.3	*P* = .013	Yes
4, subtraction	12	58.9	*P* = .074	No*

* Although performance on these trials is not significantly above chance, the mean score is above 50%. If children were using the range strategy to solve these problems performance would be significantly below chance.

Difference strategy. We tested whether children solved the addition and subtraction problems by comparing two numbers involved in the problem. In addition problems, if the difference between the comparison set and the larger of the addends is small (< 10) then the correct answer is always that the comparison set is smaller . Conversely, if the difference between the comparison set and the larger of the addends is large (> 20) then the correct answer is always that the comparison set is larger. A similar relation holds for the subtraction problems: if the difference between the comparison set and the minuend is small (< 10) then the correct answer is always that the comparison set is larger. If the difference between the comparison set and the minuend is large (> 20) then the correct answer is always that the comparison set is smaller.

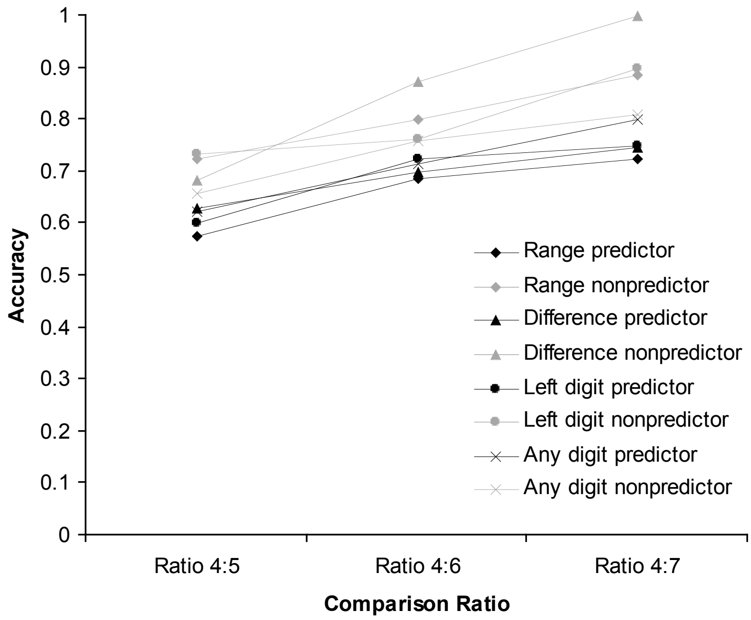
We therefore analyzed subsets of addition and subtraction problems where the critical difference was medium size (e.g. $10 < d < 20$) and the correct answer could not be reliably predicted (see Supplementary Table 4). In the first addition experiment, there was some evidence that children may have used the difference strategy, however, this effect may also be due to children's overall tendency to answer 'more' in this experiment, since the children were also at chance for problems with a large difference between the comparison set and larger addend where the difference strategy could have been used but the answer was always less (57.5%, $t_{19} = 0.90$, $P = .379$). In the other experiments, there was no evidence children based their answer on the difference between sets since they performed above chance on the critical problem subsets.

Supplementary Table 4: Performance in each experiment on the subset of problems in which the difference between the comparison set and addend / minuend is not predictive of the correct answer.

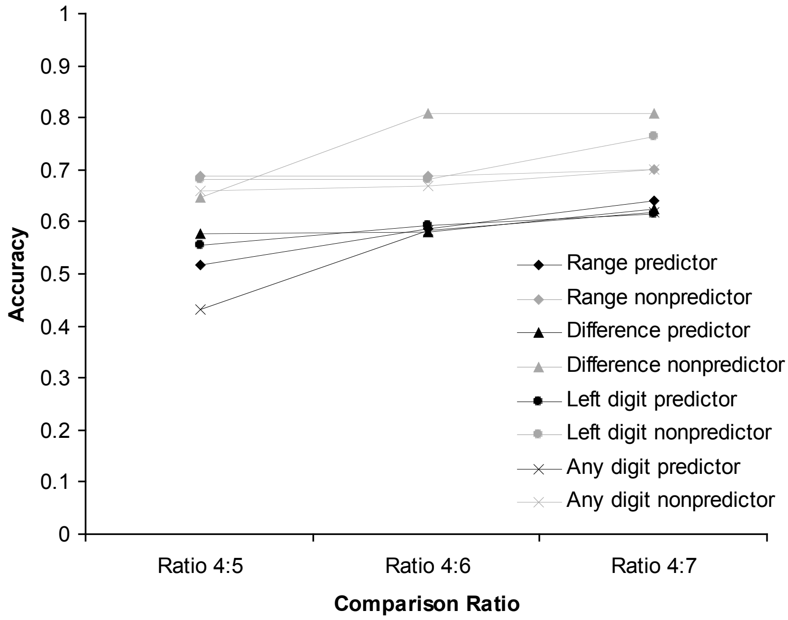
Experiment	Trials / subject	Accuracy	*P* Value	> Chance (50%)
1, addition	3	56.7	$P = .345$	No*
2, addition	7	82.9	$P = 4\text{x}10^{-11}$	Yes
3, addition	7	74.1	$P = 1\text{x}10^{-9}$	Yes
4, subtraction	7	73.5	$P = 2\text{x}10^{-6}$	Yes

* The small number of problems with a medium difference limits the extent to which the difference strategy can be tested in Experiment 1.

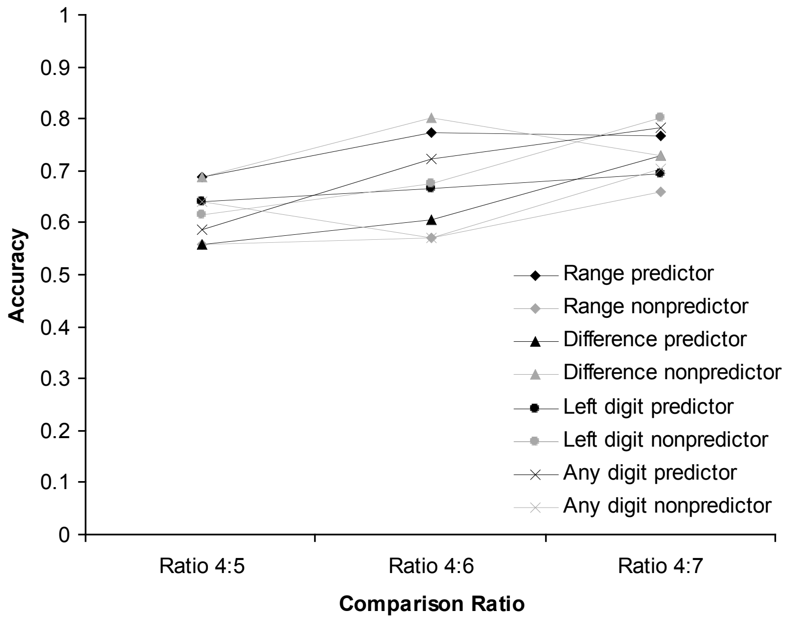
Ratio effect and strategies. A final set of analyses investigated whether the ratio effect observed in the addition and subtraction experiments (higher performance when the comparison quantity differed from the sum or difference by a ratio further from 1) might be an artefact of children's tendency to use one of the strategies described above. Because the problems on which each of these strategies could or could not be used were not always evenly distributed across the three ratios, it is possible that the ratio effect could have resulted from children's ability to use these strategies more effectively at larger ratios than at smaller ratios. To investigate this possibility, performance at each ratio was graphed separately for problems on which each alternative strategy could or could not be used. These figures demonstrate that the linear trend is observed for all of these subsets of problems, and thus the overall ratio effects did not arise simply due to use of these strategies. To examine whether the linear effect of ratio was the same across problems that were predictors and non-predictors for each strategy, three-way ANOVAs (ratio x strategy x predictor/nonpredictor) were performed. There was no interaction between ratio and predictor status for any of the experiments (addition lab $F_{2,38} = .774$, $P = .468$; addition classroom $F_{2,72} = .225$, $P = .799$; subtraction $F_{2,54} = .983$, $P = .381$).



Supplementary Figure 2: Laboratory-based addition performance at each ratio for problems on which different strategies can (predictor) and cannot (nonpredictor) be used.



Supplementary Figure 3: School-based addition performance at each ratio for problems on which different strategies can (predictor) and cannot (nonpredictor) be used.



Supplementary Figure 4: Subtraction performance at each ratio for problems on which different strategies can (predictor) and cannot (nonpredictor) be used.

Supplementary Table 5: Problem sets used in each experiment.

Verbal Addition	Computer-based Addition	Comparison	Computer-based Subtraction
Ratio 4:6	Ratio 4:5	Ratio 4:5	Ratio 4:5
28 + 16 vs. 65	6 + 6 vs. 15*‡	12 vs. 15	20 – 8 vs. 15*‡
34 + 35 vs. 46†	9 + 6 vs. 12‡	20 vs. 16	24 – 9 vs. 12†‡
17 + 5 vs. 32*†‡	12 + 8 vs. 16	45 vs. 36	25 – 5 vs. 16†‡
24 + 27 vs. 35	7 + 9 vs. 20*†‡	40 vs. 50	23 – 7 vs. 20*‡
48 + 18 vs. 98	25 + 20 vs. 36*†		58 – 13 vs. 36*
13 + 14 vs. 18	20 + 16 vs. 45	Ratio 4:6	61 – 25 vs. 45†‡
32 + 25 vs. 38	15 + 25 vs. 50	10 vs. 15	66 – 26 vs. 50†
5 + 19 vs. 36*†‡	20 + 30 vs. 40*†	14 vs. 21	63 – 14 vs. 40*
		45 vs. 30	
	Ratio 4:6	51 vs. 34	Ratio 4:6
	5 + 5 vs. 15*†‡		17 – 7 vs. 15*‡
	9 + 6 vs. 10‡	Ratio 4:7	21 – 6 vs. 10†‡
	8 + 6 vs. 21*†‡	21 vs. 12	23 – 9 vs. 21*‡
	9 + 12 vs. 14‡	13 vs. 23	27 – 6 vs. 14†‡
	25 + 20 vs. 30*	32 vs. 56	57 – 12 vs. 30*
	15 + 15 vs. 45	58 vs. 33	48 – 18 vs. 45
	21 + 30 vs. 34*		64 – 13 vs. 34*
	15 + 19 vs. 51		55 – 21 vs. 51
	Ratio 4:7		Ratio 4:7
	10 + 11 vs. 12‡		30 – 9 vs. 12‡
	6 + 6 vs. 21*†‡		23 – 11 vs. 21*
	6 + 7 vs. 23*†‡		24 – 11 vs. 23*
	11 + 12 vs. 13		31 – 8 vs. 13‡
	16 + 16 vs. 56		65 – 33 vs. 56†
	30 + 26 vs. 32*		70 – 14 vs. 32*
	27 + 31 vs. 33*		71 – 13 vs. 33*
	16 + 17 vs. 58		70 – 37 vs. 58†

*** Problems for which the range strategy is not a predictor**

† Problems for which the difference strategy is not a predictor

‡ Problems for which the single-digit strategies are not predictors

Reference

Barrouillet, P. & Lépine, R. Working memory and children's use of retrieval to solve addition problems. *J. Exp. Child Psychol*. 91, 183-204 (2005).